Money and Economic Growth Revisited: A Note

Eric Kam
Associate Professor of Economics, Ryerson University
erickam@economics.ryerson.ca

and

John Smithin
Professor Emeritus of Economics and Senior Scholar, York University
Executive Co-Director and Fellow, Aurora Philosophy Institute
jsmithin@yorku.ca

Abstract

In an important, but often neglected, paper written four decades ago at a critical juncture in the development of the history of monetary and macroeconomic thought, Begg (1980) attempted to solve the theoretically ambiguous puzzle of whether monetary policy is super-neutral in the steady-state. The proposition of monetary super-neutrality was shown to depend crucially on two sufficient conditions, only one of which is necessary. Begg argued that a more general specification, which violates both sufficient conditions, would restore the argument for monetary *non*-super-neutrality. This note suggests that there is one additional sufficient condition to yield monetary super-neutrality in the class of models described by Begg. The demand for real balances must be modeled as a decreasing function of the real, not the nominal, rate of interest. The addition of this sufficient condition does not necessarily nullify Begg's criticisms of the rational expectations school. However, modeling the demand for real balances as a function of the real rate of interest rate does have implications for models that assume a steady-state. Harkness (1978) had previously shown that this extra sufficient condition for monetary super-neutrality is simultaneously a *necessary* condition for the existence of the steady-state.

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Introduction

In an important but often neglected paper in the *Review of Economic Studies*, written at a critical juncture in the development of the history of monetary and macroeconomic thought in the twentieth century, Begg (1980) attempted to solve the theoretically ambiguous puzzle of whether monetary policy is super-neutral in the steady state. The proposition of monetary super-neutrality, taken for granted in the rational expectations literature of the time, was shown to depend crucially on two sufficient conditions, only one of which is necessary. Begg argued that a more general specification, which violates both sufficient conditions, would restore the argument for monetary *non*-super-neutrality as put forward earlier by such writers as Mundell (1963) and Tobin (1965).

This note argues there is an additional sufficient condition to yield monetary super-neutrality in the class of models described by Begg. Money is also super-neutral if the demand for real balances is modeled as a decreasing function of the real, not the nominal, rate of interest. This possibility was never considered in the macroeconomic literature of the period. However, it certainly should have been taken into account because, two years before the publication of Begg's paper, Harkness (1978) in a paper published in the *Canadian Journal of Economics*,[1] had shown that this additional sufficient condition for monetary super-neutrality is simultaneously a *necessary* condition for the existence of the steady-state in neoclassical-type models.

The argument does not end the debate over whether monetary policy is, in fact, neutral or super-neutral. At the time of these discussions in the twentieth policy was conceived of very naively as entailing only deliberate changes in the rate of growth of the money supply. No attention was paid to such issues as endogenous money, endogenous time preference, the rate of interest rate as a policy instrument, or even to the basic ontological question of the existence or non-

[1] For whatever reason, this issue always seems to have has actively interested Canadian and Canadian-based economists, in particular.

existence of a 'natural rate' of interest. A more careful consideration of any or all of these issues would clearly have the potential to affect the results dramatically (Kam 2000, 2005, Reis 2007, Smithin 2013, 2018, Kam, Smithin and Tabassum 2019). However, Harkness's proof *is* decisive for the entire class of models that were under active discussion at the time. In effect, a presumption of super-neutrality was built into the basic premises of the mid-twentieth literature.

Section 1 describes Begg's two simultaneous conditions for steady-state monetary super-neutrality, and Section 2 constructs a macroeconomic model to formally derive these conditions. Section 3 derives the third sufficient condition, and the concluding section offers a summary.

1. Sufficient Conditions for Monetary Super-Neutrality

Monetary super-neutrality exists if changes in the rate of growth of the money growth rate of the money supply do not affect any of the real sector variables in a macroeconomic system (Sidrauski 1967). Begg revealed an apparent inconsistency in this respect between two competing historical literatures; the steady-state analysis of macroeconomic models featuring rational expectations (Lucas 1972, Sargent and Wallace 1975, Barro 1976), and an earlier literature on the effect of monetary policy in the neoclassical growth model (Mundell 1963, Tobin 1965, Johnson 1967). The rational expectations literature largely portrays monetary policy as super-neutral in the steady state, whereas the neoclassical 'money and growth model' typically described monetary policy as *non*-super-neutral in the steady state. Begg tried to solve this theoretical puzzle by deriving a general specification which violates monetary super-neutrality in the steady-state.

Begg's argument did not contradict one of the central features of the rational expectations framework, namely the principle that 'in a steady state, *any* expectations generating mechanism will yield correct predictions' (Begg 1980, 293 emphasis added). Therefore, the money and growth model should properly be interpreted as a special case of the rational expectations

framework which makes the inconsistency between the two literatures the more surprising. Begg's conclusion was that rational expectations models lacked one important feature of monetary growth models, the presence of a real balance effect in the consumption function. He argued that if there is real balance effect in the consumption function, both rational expectations and monetary growth models produce monetary *non*-super-neutrality.

2. A Simple Formal Model

In a notation first employed by Smithin (1980), Begg's argument can be reproduced as;

(1) $y = c(y, w) + dk/dt + dk,$ $\quad 0 < c_y < 1,\ c_w > 0$

(2) $m = l(y, i),$ $\quad l_y > 0,\ l_i < 0$

(3) $y = f(k)$ $\quad fk > 0,\ f_{kk} < 0$

(4) $w = k + m$

(5) $i = r + \pi$

(6) $r = f_k - \delta$

(7) $dm/dt = m(\theta - \pi).$

Here y stands for real output, c is the consumption function, w is total real wealth, k is the capital stock, m is the real stock of money, l is the liquidity preference function, f is the production function, i is the nominal interest rate, r is the real rate of interest , and θ is the rate of monetary growth. The symbols π, δ, and θ stand for the inflation rate, the depreciation rate, and the rate of monetary growth, respectively. By definition in the steady-state;

(8) $dk/dt = dm/dt = 0$

The steady-state capital stock k^* can therefore found by solving the following equation,[2]

[2] As is well-known, from the formal point of view the main analytical result of the neoclassical growth model was simply to solve for the conditions of equilibrium in the capital market. The growth rate itself was supposed to be

(9) $f(k^*) = c\{f[k^*], k^* + l[f(k^*), fk(k^*) - \delta - \theta)\} + dk^*.$

And, the effect of a change in the rate of monetary growth, θ, on k^* is given by;

(10) $dk^*/d\theta = c_w l_i/[f_k(1 - c_y - c_w l_y) - c_w(1 + l_i f_{kk}) - \delta]$

If money is super-neutral, $dk^*/d\theta = 0$, Therefore, the two alternative sufficient conditions for monetary super-neutrality are;

(11) $c_w = 0$

(12) $li = 0$

The first sufficient condition implies the absence of a wealth effect in the consumption function while the second eliminates the interest rate/opportunity cost term in the demand for money function. Begg (1980, 296) rejects the first condition, and dismisses the second on the grounds that it ' ... effectively denies Keynesian liquidity preference'.

As it stands in equation (10), the sign of $dk^*/d\theta$ is ambiguous, but Begg resolves this by examining the dynamic properties of the system around the steady-state and appealing to Samuelson's (1983) correspondence principle. If the system is to be 'saddle-point stable' (Sargent 1973), then $dk^*/d\theta$ must be positive.

3. A Third Sufficient Condition

The upshot of Harkness's (1978) argument, about the existence conditions for a neoclassical steady-state with money, was to bring into question the conventional specification of the demand for money function. The conventional specification was that each agent perceives the return on real balances to be negative and equal to the inflation rate. The opportunity cost of holding real

determined by exogenous factors, such as technical progress or the rate of growth of the labour force.

balances is therefore thought to be the sum of the rate of return foregone by not holding other assets plus the inflation rate. If λ stands for the opportunity cost of holding real money balances;

(13) $$\lambda = r - (-\pi) = r + \pi$$

However, Harkness went into this question more deeply by examining the existence of the steady-state in a typical neoclassical monetary growth model, in which new money is injected into the economy *via* direct transfer payments to economic agents. The transfers can either be completely random in the sense that they are not related to each individual's initial money holdings - as in Friedman's (1969, 5) famous 'helicopter money' - or they may be tied to initial holdings at a rate which may, or may not, differ across individuals.

But, Harkness (1978, 11) was able to demonstrate that a random distribution 'is inconsistent with the existence of stable steady-equilibrium'. And the only transfer rate that will avoid 'distributional effects' Harkness (1978, 705) - which would also disturb the equilibrium - is a rate which is itself the same for all individuals and (adjusted for population growth) is equal to the rate of monetary growth. What then is the opportunity cost of holding real money balances in such a world (Smithin 1983, 68)? If we let τ stand for the transfer rate then, instead of the formulation in (13), the opportunity cost of holding real money balances now turns out to be;

(14) $$\lambda = r + \pi - \tau$$

In the Begg model the rate of population growth is zero which means that the transfer rate must be $\tau = \theta$. Also, in equilibrium the inflation rate will be given by $\pi = \theta$. Therefore, the opportunity cost of holding real balances is;

(15) $$\lambda = r + \theta - \theta = r.$$

The opportunity cost of holding real balances is therefore simply the real rate of return to be earned by holding other assets, and this will be the appropriate argument in the demand for

money function. Replacing the demand for money function $m = l(y, i)$ with $m = l(y, r)$, where $ly > 0$ and $lr < 0$, we can thus modify equation (9) above to now read;

(16) $f(k^*) = c\{f[k^*], k^* + l[f(k^*), i - \theta)\} + \delta k^*.$

Totally differentiating;

(17) $fkdk^* = cyfkdk^* + cwlrdk^* + cwlrfk + cwlrdi - cwlrdq + ddk^*.$

From equation (2), in the steady-state we also have;

(18) $di = f_{kk}dk^* - \delta dk^* + d\theta$

And, finally, substituting (18) into (17);

(19) $dk^*/d\theta = (cwl_r - c_w l_r)/[f_k(1 - c_y - c_w l_y) - c_w(1 + l_r f_{kk})] = 0.$

This is therefore a third sufficient condition which guarantees super-neutrality regardless of the values of c_w and l_r.

Conclusion

Because of their common roots in the neoclassical growth model neither the money and growth literature, nor rational expectations models, are able to account for monetary *non*-super-neutrality. This was a fatal flaw of the mid-twentieth literature. In spite of the claims of writers such as Mundell and Tobin, and later Begg, it is not possible to restore the idea of 'forced saving', as the phenomenon was known in the history of economic thought (Hayek 1932), within that framework. Such a project would require a much more sophisticated and factually based monetary theory than existed in the mid-to-late 20th century, involving such things as endogenous money, an interest rate policy instrument, and a careful re-consideration of the relationship between the central bank and the commercial banks in the monetary policy transmissions mechanism (Kam and Smithin 2012, Smithin 2020).

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