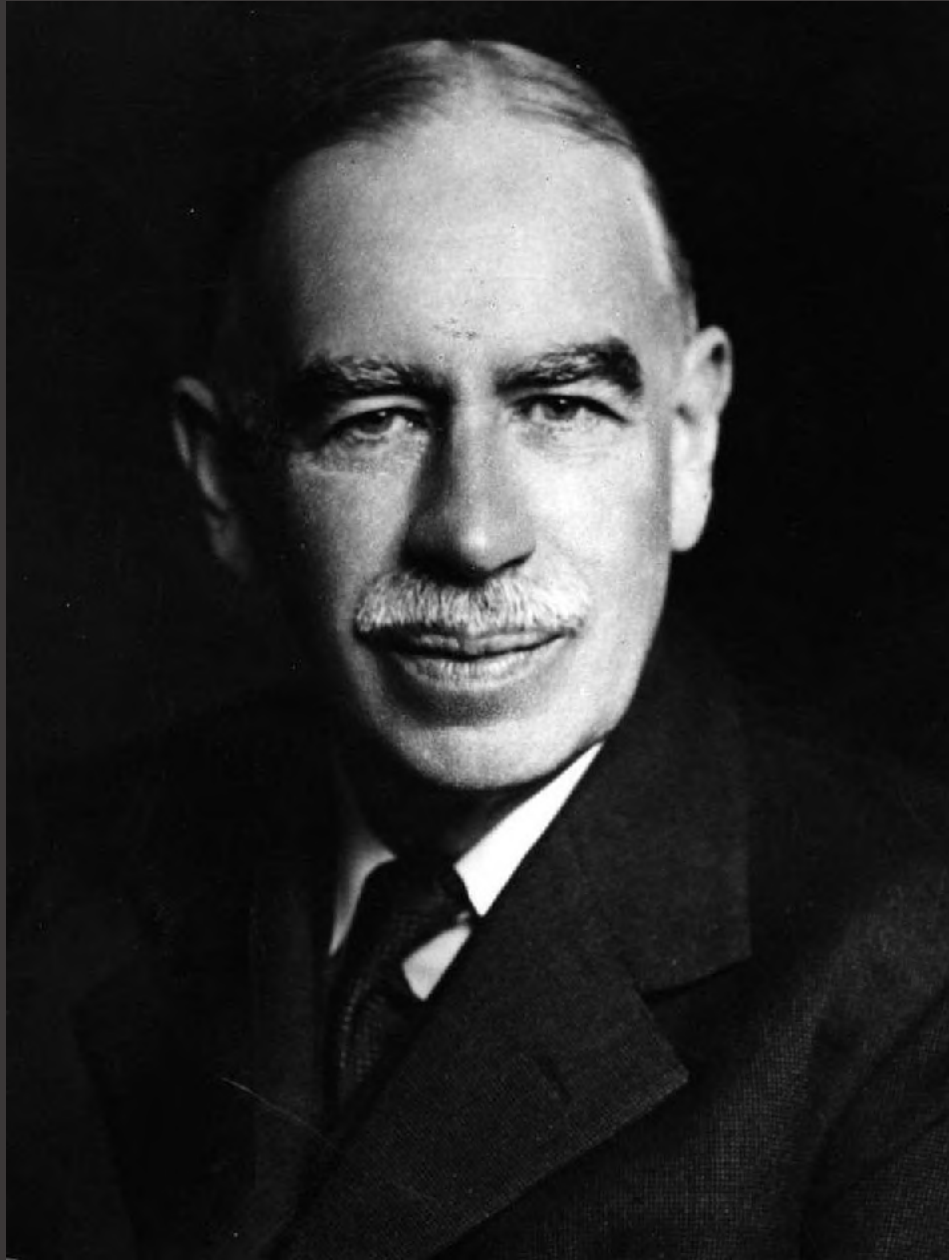# Two searching examinations of randomness and limited knowledge in 1921

- John Maynard Keynes, *A Treatise on Probability*

- Frank H. Knight, *Risk, Uncertainty, and Profit*

- Contrasting appearance: one on philosophical foundations of probability (first in English for 55 years), other on a theory of profit as payment for bearing uncertainty

- But shared a central concern with the distinction between randomness that could be represented by a probability distribution and hence was insurable (risk) and randomness that could not be so represented (uncertainty)

- Terms risk and uncertainty used in Knight (1921) and in Keynes, *General Theory* (1936) -- but Keynes (1921), although using those concepts, did not use those terms

- Acknowledged different intellectual influences: for Keynes, Cambridge philosophers W. E. Johnson, G. E. Moore, Bertrand Russell

- For Knight, Iowa colleague Charles O. Hardy and Cornell and Chicago economists Alvin Johnson, Allyn Young, John Maurice Clark, Jacob Viner (but also cited John Neville Keynes)

- Both books were revisions of prewar dissertations: Keynes for fellowship at King's College, Cambridge (unsuccessful Dec. 1907, successful Dec. 1908), Knight for PhD at Cornell 1916 (prewar for US), then Hart, Schaffner & Marx essay competition (2nd prize 1917)

- Keynes added last 7 (of 33) chapters later, on statistical inference, developing arguments in dissertation – set up in proof 1913 but many later corrections to proofs – paid £767 11s to Macmillan for 2,500 copies

- Two of the three great pre-World War I dissertation on randomness in economics, third being Louis Bachelier (1900) on random walks and Brownian motion in efficient asset markets – cf. Keynes's *JRSS* review of Bachelier (1912)

- Each of these pathbreaking books of 1921 has generated its own literature

- Some scholars have attended closely to parallels and differences between the 2 books (e. g. a book by W. B. Greer 2001)

- Knight (1921) and Keynes (1921) share a chapter ("The Radically Distinct Notion") in Peter Bernstein, *Against the Gods: The Remarkable Story of Risk* (1998), for more general audience

- But the two literatures have engaged with each other only intermittently

- Knight's name does not appear in such books as Alan Coddington, *Keynesian Economics: The Search for First Principles* (1983) or Roger Backhouse and Bradley Bateman, eds., *The Cambridge Companion to Keynes* (2006) – or, regrettably, in Dimand and Harald Hagemann, eds., *The Elgar Companion to John Maynard Keynes* (2019)

- Moggridge (1992) and Skidelsky (2003), both attentive to the role of uncertainty in Keynes's thought, mention Knight only for his caustic review of *The General Theory*

- Key contributions on Knightian uncertainty and Knightian decision theory by Truman Bewley and by Kiyohiko Nishimura and Hiroyuki Ozaki (on both of whom more later) do not mention Keynes

- Knight notoriously hostile to Keynes's *General Theory*

- According to Keynes, the 2 conclusions of Knight's review: "namely, that my book caused him intense irritation, and that he had great difficulty in understanding it"

- Knight, in AEA presidential address at end of 1950: "The latest 'new economics,' and in my opinion rather the worst for fallacious doctrine and pernicious consequences, is that launched by the late John Maynard (Lord) Keynes, who for a decade succeeded in carrying economic thinking well back to the dark age; but of late this wave of the future has happily been passing"

- Fairly moderate language for a presidential address in which Knight likened the Pope to Hitler and Stalin

- Yet, despite differences between Keynes and Knight on other matters, and despite differences in style and in intellectual origins between their 1921 books, the concept of fundamental, uninsurable uncertainty stems from both Keynes (1921) and Knight (1921) and the two resulting literatures have important things to say to each other – not just for getting the history in balance but for understanding the place of uncertainty in economics

- In particular I shall argue that contributions on Knightian decision theory by Truman Bewley and by Nishimura and Ozaki offer a response to a formidable critique of Keynesian uncertainty by Alan Coddington

- Coddington, "Deficient Foresight: A Troublesome Theme in Keynesian Economics" (*AER* 1982) and *Keynesian Economics* (1983)

- Bewley, "Knightian Uncertainty" ([1988] 1998) in Jacobs et al., eds., *Frontiers of Research in Economic Theory: The Nancy Schwartz Memorial Lectures,* Econometric Society Monograph (nonmathematical statement)

- Bewley, "Knightian Decision Theory and Econometric Inferences," Cowles Foundation Discussion Paper (1988), *Journal of Economic Theory* (2011)

- Nishimura and Ozaki, *Economics of Pessimism and Optimism: Theory of Knightian Uncertainty and Its Applications* (Springer 2017) – incorporates *Journal of Economic Theory* articles from 2004, 2007, 2015 with "Knightian uncertainty" in title

- A note of explanation: I am by no means part of the normal target audience of *JET* or books based on *JET* articles, but I have known Kiyohiko Nishimura since we entered grad school at Yale in 1978; at his suggestion, I was invited to review the 2017 book in *Journal of Economics/Zeitschrift für Nationalökonomie*; since 1983 Truman Bewley has been at the Cowles Foundation (Yale), of which I am writing a history

- Perhaps a bit surprising that neither Bewley nor Nishimura thought of Keynes in connection with fundamental uncertainty, since both known for work on microeconomic foundations of New Keynesian macro:

- Nishimura, *Imperfect Competition, Differential Information, and Microfoundations of Macroeconomics* (OUP 1992)

- Bewley, *Why Wages Don't Fall During a Recession* (Harvard UP, 1999)

but New Keynesian economists don't read Keynes, hence bits of Keynes keep being independently reinvented (e. g. relative wage model of Keynes 1936, Ch. 2, as Taylor staggered wage model) – also Nishimura and Ozaki seem not to have been aware of Bewley's papers

- In contrast to frequentist theory of probability (Richard von Mises), which saw probabilities as empirical regularities, for Keynes (1921), "The Theory of Probability deals with the relation between two sets of propositions … perception of which, together with knowledge of the first set, justifies an appropriate degree of rational belief about the second"

- Keynes (1921): "The perceptions of some relations of probability may be outside the powers of some or all of us. What we know and what probability we can attribute to our rational beliefs is, therefore, subjective in the sense of being relative to the individual. But given the body of premises which our subjective powers and circumstances supply to us, and given the kinds of logical relations, upon which arguments can be based and which we have the capacity of perceive, the conclusions, which it is rational for us to draw, stand to these premises in an objective and wholly logical relation."

- Concession that perception of some logical relations of probability may be beyond the powers of "all of us" helped make Keynes (1921) vulnerable to subjectivist critique by Frank Ramsey

- Ramsey, prodigy, Senior Wrangler, reviewed Keynes (1921) in *Cambridge Magazine* at age 19; also paper on "Truth and Probability" to Cambridge Moral Sciences Club (1926), published in posthumous 1931 volume of his essays (died at age 26)

- Much has been written about the extent to which Keynes accepted Ramsey's critique in his memorial article on Ramsey (reprinted in *Essays in Biography*) and hence extent to which *Treatise on Probability* is a guide to probability, expectation and uncertainty in *General Theory*

- Books by Bradley Bateman (1996) and John Davis (1994) argue for a discontinuity in Keynes's view of probability:

- "But the basis of our degrees of belief – or the *a priori* probabilities, as they used to be called – is part of our human outfit, perhaps given us merely by natural selection, analogous to our perceptions and our memories rather than to formal logic. So far I yield to Ramsey – I think he is right."

- But Keynes immediately continued, "But in attempting to distinguish 'rational' degrees of belief from belief in general he was not yet, I think, quite successful. It is not getting to the bottom of the problem of induction merely to say it is a useful habit"

- Books by Anna Carabelli (1988) and Rod O'Donnell (1989) argue for continuity in Keynes's view of probability

- *Treatise on Probability* (Ch. 6, "The Weight of Arguments") cited only once in *General Theory* (p. 148) – but citation is regarding how certain or confident the state of long-term expectation is, so *TP* relevant for view of uncertainty

- Why does it matter?

- Because Ramsey's (and Bruno De Finetti's) view of subjective probability leads to the axiomatization of subjective probability by Leonard Jimmie Savage, which has no place for the distinction between risk and uncertainty

- G. L. S. Shackle, Brian Loasby, Jan Kregel (1976), E. Roy Weintraub (1975) drew renewed attention to claims by Keynes (1st half of 1937 *QJE* response to reviewers), Hugh Townshend (1937), Shackle (1938) that fundamental uncertainty is a (and arguably the) crucial element of the central message of *The General Theory*

- Keynes (1937): "The sense in which I am using the term [uncertainty] is that in which the prospect of a European war is uncertain … About these matters there is no scientific basis on which to form any calculable probability whatever" so depend on rules of thumb, animal spirits

- Black swan (Nassim Nicholas Taleb) – potential surprise (Shackle) – cannot even list all possible outcomes

- Fundamental limitation of knowledge, whether because world is inherently unpredictable (nonergodic in Paul Davidson's term) or epistemic limitations of humans or because humans recognize free will of other humans (Greer's interpretation of Knight)

- Keynes invoked fundamental uncertainty to argue that private investment decisions are volatile, implying role for offsetting aggregate demand policy

- but Hayek (essays collected as *Individualism and the Economic Order*, 1948) reached very different policy conclusions by emphasizing that limited knowledge, fundamental uncertainty affect all decision-making, including public policy

- Foundations of Bayesian subjective probability theory sufficiently contested so that I. J. Good, addressing a conference on foundations of probability theory, referred to distinction in literature between "Savage Bayesians and Good Bayesians" and said he was going to have to refer to "sweeping under the carpet" so often that "I shall use the abbreviations UTC and SUTC"

- But all varieties of Bayesian decision theory share the disappearance of fundamental uncertainty

- Instead of being uncertain to a greater or less degree, individuals form a subjective probability distribution by attaching a weight to each possibly true probability distribution

Stigler's unease with his mentor's thesis

- Hence George Stigler's 1971 introduction to 50th anniversary edition of Knight (1921):

- "Tradition has assigned a distinction between risks (capable of actuarial treatment) and uncertainty (stochastic events not capable of such treatment) as Knight's contribution. Fortunately this is an extreme caricature of his work, because modern analysis no longer views the two classes as different in kind"

- Does not affect "Knight's famous result" that pure profit is the difference between payments to all hired factors (including risk premia) and the realized profit – trivializing "Knight's famous result" as a mere definition, and ignoring the prospect that, unless markets mispriced risk, risk premia would eliminate pure profit

Coddington on deficient foresight as a troublesome theme in Keynesian economics

- Invoking erasure of risk/uncertainty distinction in subjective probability theory, Coddington called "into question the idea that there is anything peculiarly subversive in the analytical consequences of broaching problems of uncertainty in economic decision-making. I shall argue, rather, that, as an analytical issue it is – depending on how it is handled – either innocuous or else quite indiscriminately destructive."

- Innocuous if reducible to risk by having a subjective probability distribution over what the true probability distribution is

- Indiscriminately destructive if all decision-making undermined by unpredictability – Keynes wrote to go from first half of 1937 *QJE* article rejecting pretense of knowledge – "We simply do not know" – to comparative static exercises, assuming stable consumption function and multiplier, in second half of article

- "Those who have pointed to this article as an interpretive key owe us, at the very least, an explanation of why, immediately after having provided his key proposition, Keynes engages in analysis in flagrant contradiction with it"

A Knightian response to Coddington's challenge

Was Coddington correct that either fundamental uncertainty must be rendered innocuous by reducing it to risk or else it is "quite indiscriminately destructive," preventing economists from knowing anything about the economy?

Some contributions to Knightian decision theory imply that Coddington was not correct (and nor was Stigler in trying to dissociate Knight from uncertainty)

- LeRoy and Singell (1987) reconciled Knight to Bayesian decision theory by reinterpreting Knight (1921) as being about market failure due to moral hazard and adverse selection, as in later economics of information, not really about uncertainty

- Without quarrelling about what Knight might have really meant, Truman Bewley, in a four papers first circulated from 1986 to 1988, presented a more general Knightian decision theory in which Bayesian decision theory is the special case in which everyone happens to be uncertainty-neutral (ambiguity-neutral)

- Like Nishimura and Ozaki, and earlier Itzhak Gilboa and David Schmeidler, Bewley dropped Kolmogorov's additivity axiom

- Unless uncertainty-neutral, cannot form a subjective probability distribution by linear combination of the possible probability distributions – have convex set of probability distributions, not additivity

- Disposes of the first of Coddington's two alternatives, the widely-accepted view that uncertainty is innocuous because same as risk

- Bayesian decision theory is special case of Knightian decision theory where everyone is uncertainty-neutral (ambiguity-neutral)

- To say that uncertainty does not matter if everyone is uncertainty-neutral is as true, and as uninteresting, as saying that risk does not matter if everyone is risk-neutral

- Bewley: "the Knightian picture of the world is one where agents operate in an environment where much of the risk in unevaluatable and therefore uninsurable. Uncertainty aversion inhibits the development of futures markets, which could themselves help reduce the uncertainty of the environment"

- Bewley's insight that uncertainty aversion inhibits development of full set of futures markets offers a response to Coddington's second alternative – economists do have something to say about an uncertain world

- Another example of a concrete implication of uncertainty: Nishimura and Ozaki (*JET*, 2004) found that increased risk and increased uncertainty (expansion of the decision-maker's set of probability distributions) have opposite effects on reservation wage in a job search model – the finding that motivated their research program on Knightian uncertainty

- Nishimura and Ozaki define a capacity as "'something' that looks like a subjective probability, but is *not* exactly a subjective probability because it does not necessarily satisfy the additivity axiom among Kolmogorov's axioms ... where people are 'uncertainty-averse,' the capacity characterizing their view on uncertainty is still nonadditive, but it obeys some strong law or pattern about the way of its being nonadditive, called *convexity* ... Roughly speaking, a capacity is *convex* if the capacity of the union of the mutually disjoint events is *larger* than or equal to the sum of the capacities of each single event"

- Convexity instead of additivity also central to Bewley's approach

- Nishimura and Ozaki (2017): "Until the global financial crisis, applications of Knightian uncertainty were relatively limited in scope, and were motivated more by theoretical curiosity and the desire to amend apparent limitations of economic theory in explaining anomalies often found in peripheral phenomena. The situation has changed drastically since the global financial crisis, and there has been a vast expansion of the Knightian uncertainty literature that has been impossible to monitor fully … the global financial crisis … largely unexpected, shaped the financial landscape for the entire tenure of one of the coauthors (Nishimura) as deputy Governor of the Bank of Japan"

Keynes and Bachelier: a missed opportunity

- The third great pre-World War I dissertation on risk and uncertainty was Louis Bachelier's 1900 Paris math thesis on the theory of speculation – little noticed at the time, now lauded in math finance as origin of martingales, random walks (discrete time) and Brownian motion (continuous time), Gaussian stochastic processes for asset prices, five years about Albert Einstein independently formalized Brownian motion for physical particles

- From principle that "The mathematical expectation of the speculator is zero" derived what, in later terminology (Fama), is concept of efficient market

- Continued to apply calculus of probabilities to financial speculation for four decades (see Bachelier 1938, translated in Ben-El-Mechaiekh and Dimand 2018)

- Anticipated by nonmathematical book by Jules Regnault (1863), Bachelier (1900) is the origin of the mathematical version of the Gaussian approach to financial mathematics antithetical to the emphasis of Keynes and Knight on uncertainty, Mandelbrot on fat-tailed distributions and "wild probability", Taleb on black swans

- Keynes (*JRSS*, 1913) reviewed Bachelier (1912) and other books on probability in French and German, including ones by A. Markoff (of Markov chains, Markov processes) and Bachelier's teacher Henri Poincaré

- Keynes (1921) cited Bachelier (1912) four times, and listed both it and Bachelier (1914) in bibliography

- But never even mentioned that Bachelier (1912) included three chapters on applications to financial speculation, summarizing and extending Bachelier (1900)

- Keynes may have stopped reading Bachelier (1912) after early chapters showed him that it was a frequentist work, concerned with empirical regularities rather than with weight of argument

- Hence also missed what Mandelbrot calls "a second but little-known achievement of Bachelier [in Bachelier 1914] … he pioneered both in discovering the Gaussian random-walk model [in 1900] and in noting its major weaknesses [in 1914] … he observed that if the sample histograms are relative to mixtures of distinct populations, their tails could be expected to be fatter than in the Gaussian case. Second, Bachelier noted that no reasonable mixture of Gaussian distributions could account for the sizes of the very largest price changes"

Keynes and Knight

- By time of Knight's review of Keynes (1936), Knight's intellectual interests had moved on from uncertainty – Keynes (1936) adopted terms risk and uncertainty, with one reference to Keynes (1921) but none to Knight (1921)

- Keynes (1936) cited Knight only for a 1934 contribution to Knight's debate with Hayek and others about Austrian theory of capital and interest, a debate of which Keynes wanted no part

- But Keynes (1921) and Knight (1921) stand as the two sources of fundamental uncertainty, generating two literatures that engage with each other only intermittently

- But, apart from achieving historical balance by seeing Keynes (1921) and Knight (1921) both introducing uncertainty despite very different styles of the two books, the two literatures can benefit from communicating and engaging with each other

- Have offered as an example of this how two contributions on Knightian decision theory (Bewley, Nishimura and Ozaki) suggest response to Coddington's challenge to Keynesian uncertainty:

- Disappearance of risk/uncertainty distinction in Bayesian decision theory is just because Bayesian decision theory is a special case where everyone is assumed to be uncertainty-neutral – otherwise, drop additivity axiom – so uncertainty not innocuous

- Also suggest responses to Coddington's second alternative: uncertainty is not "quite indiscriminately destructive" – economists can still have things to say without the additivity axiom

- E. g. Nishimura and Ozaki's 2004 finding that increased risk and increased uncertainty have opposite effects on reservation wage in a job search model

- Centenary of these two great works offers an occasion for bringing two literatures together for their mutual benefit